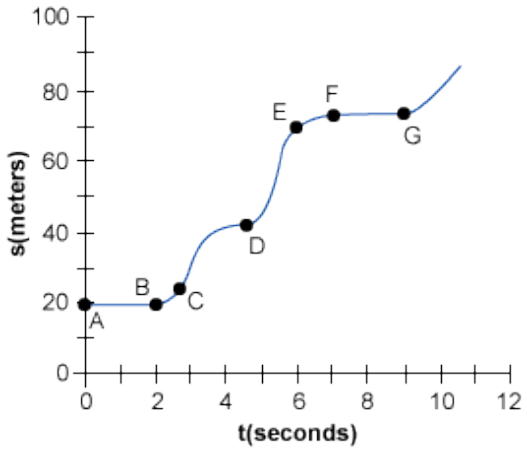
AP Calculus AB
Test: Derivatives
Instructor's Version

Free-Response Question 1—no calculator (worth 15 points)

1. The graph below shows the position function, *s*, of a car at time *t*. Use the graph to answer the following questions. Justify your answers in parts A through D.



A. What was the initial velocity of the car?

Answer: The initial velocity was zero, because the slope of the tangent line at $t=0$ is zero. (1 point for slope; 2 points for valid justification)

B. Was the car going faster at C or at D?

Answer: The car was going faster at C, since the slope of the tangent at C is greater (steeper) than the slope of the tangent at D.
(2 points for correct answer; 2 points for valid justification)

C. Was the car speeding up or slowing down at E?

Answer: The car was slowing down at E, because the tangent lines near E are becoming less steep. (2 points for correct answer; 2 points for valid justification)

D. What happened between F and G?

Answer: The car did not move during the time from F to G, because the slope of the tangent line is zero, indicating the car's velocity was zero during that time. (2 points for slope; 2 points for valid justification)

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2. The displacement, in feet, of a particle moving along the *x*-axis is given by the equation $s(t) = 2t^3 - 5t^2 - 3$, where *t* is measured in seconds.

A. Determine the velocity function $v(t)$ and find the velocity of the particle at the three times indicated below.

$v(t) =$

a. $t = 1$

Answer: $v(t) = s'(t) = 6t^2 - 10t$, $v(1) = -4$ ft/sec (2 points for velocity function; 1 point for answer [$\frac{1}{2}$ point only for answer if units are missing])

b. $t = 2$

Answer: $v(2) = 4$ ft/sec (1 point; $\frac{1}{2}$ point only if units are missing)

c. $t = 3$

Answer: $v(3) = 24$ ft/sec (1 point; $\frac{1}{2}$ point only if units are missing)

B. When is the particle moving to the left? Justify your answer.

Answer: The particle is moving to the left when its velocity is negative. (2 points for justification/work)

$v(t) = 6t^2 - 10t < 0$

$0 < t < \frac{5}{3}$

The particle is moving to the left on the interval (0, $\frac{5}{3}$). (2 points for interval)

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C. Find the average velocity on the time interval.

a. [2, 2.5]

Answer:
$s(2) = -7$ and $s(2.5) = -3$

$$\text{velocity} = \frac{\Delta\text{position}}{\Delta\text{time}} = \frac{-3-(-7)}{2.5-2} = 8 \text{ ft/sec}$$

(2 points for answer; $\frac{1}{2}$ point for units)

b. [t, $t+h$]

Answer:
$s(t) = 2t^3 - 5t^2 - 3$
$s(t+h) = 2(t+h)^2 - 5(t+h)^2 - 3$

$$\text{velocity} = \frac{\Delta\text{position}}{\Delta\text{time}} = \frac{s(t+h)-s(t)}{(t+h)-(t)} = 6t^2 + 6th + 2h^2 - 10t - 5h \text{ ft/sec}$$

(1 point for setting up the solution; 2 points for answer; $\frac{1}{2}$ point for units)

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